Lec 25: 11/20/2013

Cosmic Microwave Background (Cont'd):

As we saw, for the modes that are superhorizon at the time of recombination, observed temperature fluctuations are determined by the Sachs-Wolfe effect:

$$\left(\frac{\delta T}{T}\right)_{obs} = \left(\frac{\delta T}{T}\right)_{int} + \Phi \underset{\uparrow}{=} -\frac{1}{2}\left(\frac{\delta T}{T}\right)_{int}$$

in a matter-dominated universe

Modes that are subhorizon at the time of recombination entered the horizon earlier. For these modes, baryon perturbations obey the equation for a perturbed fluid, which is essentially a harmonic oscillator equation:

$$\ddot{\delta}_B + 2H(t)\,\dot{\delta}_B + \frac{k^2}{3a^2(t)}\,\delta_B - 4\pi G \rho_0\, \epsilon_{DM}\, \delta_{DM} = 0$$

The last term on the left-hand side can be read as the gravitational potential Φ, which is given by the Poisson equation in the subhorizon regime:

$$\Phi = -4\pi G \left[\frac{a(t)}{k}\right]^2 \rho_0 \, \varepsilon_{DM} \, \delta_{DM}$$

We note that the main contribution to the gravitational potential comes from dark matter since $\varepsilon_{DM} \gg \varepsilon_B$.

Equation for baryon perturbations can the be written to include Φ instead of δ_{DM}:

$$\ddot{\delta}_B + 2H(t)\,\dot{\delta}_B + \frac{k^2}{3a^2(t)}\,\delta_B + \Phi = 0$$

Due to photon-baryon coupling, we have:

$$\delta_B = \frac{\delta n_\gamma}{n_\gamma} = 3\left(\frac{\delta T}{T}\right)_{int}$$

Thus:

$$3\left(\frac{\delta T}{T}\right)''_{int} + 6H(t)\left(\frac{\delta T}{T}\right)^{\cdot}_{int} + \frac{k^2}{a^2(t)}\left(\frac{\delta T}{T}\right)_{int} + \Phi = 0$$

In terms of $\left(\frac{\delta T}{T}\right)_{obs} = \left(\frac{\delta T}{T}\right)_{int} + \Phi$, this gives us:

$$3\left(\frac{\ddot{\delta T}}{T}\right)_{obs} + 6H(t)\left(\frac{\dot{\delta T}}{T}\right)_{obs} + \frac{k^2}{a^2(t)}\left(\frac{\delta T}{T}\right)_{obs} = 0$$

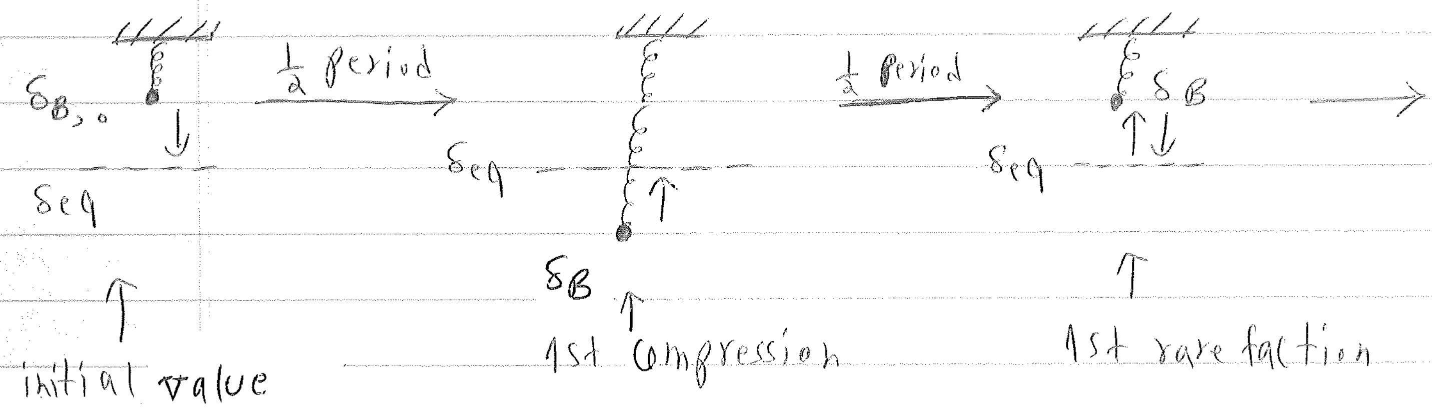
We have used the fact that $\delta_{DM} \propto t^{\frac{2}{3}}$ in the matter-dominated phase, which implies $\Phi = \text{Const}$. Finally, we arrive at:

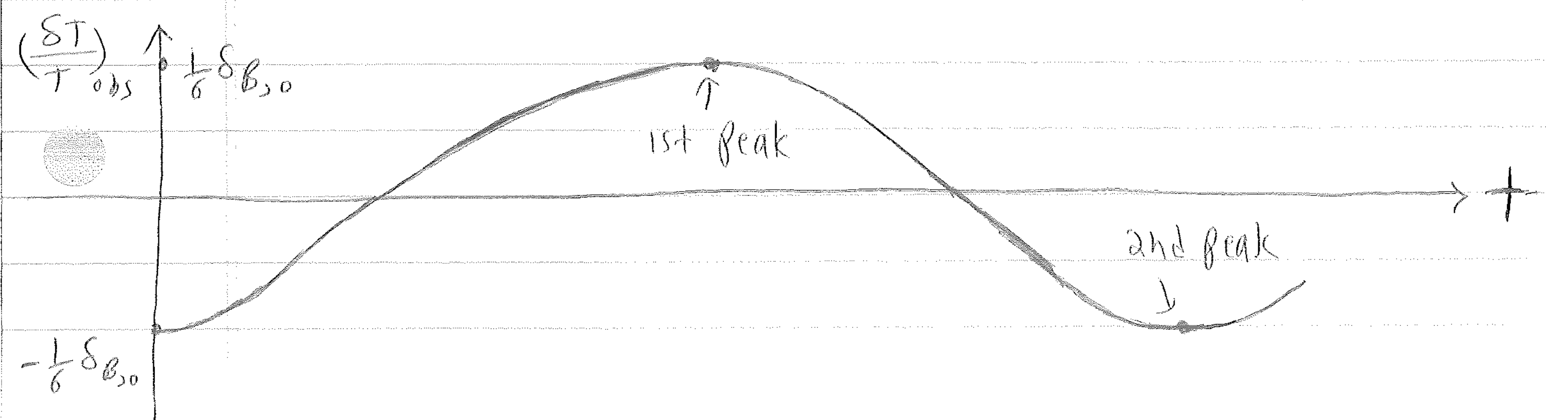
$$\ddot{\left(\frac{\delta T}{T}\right)}_{obs} + 2H(t)\,\dot{\left(\frac{\delta T}{T}\right)}_{obs} + \frac{k^2}{3a^2(t)}\left(\frac{\delta T}{T}\right)_{obs} = 0$$

Neglecting the damping term, the solution is:

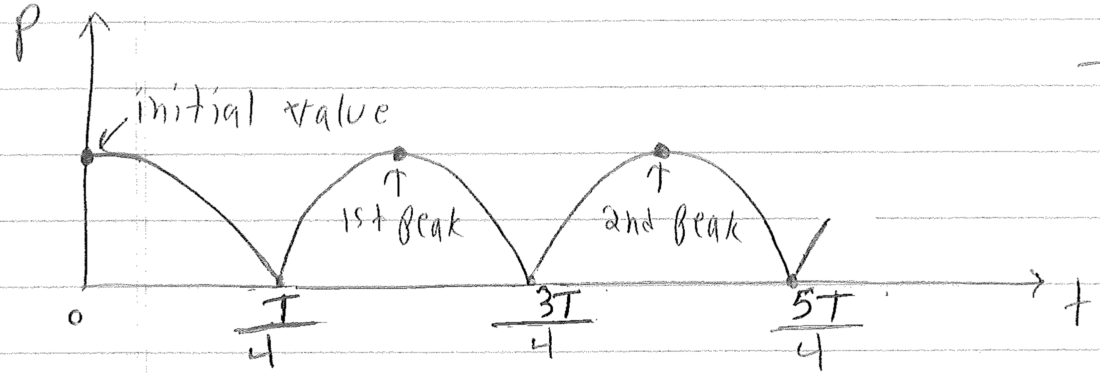
$$\left(\frac{\delta T}{T}\right)_{obs} = \left(\frac{\delta T}{T}\right)_0 \cos\omega t \qquad \omega = \frac{1}{\sqrt{3}}\frac{k}{a},\ \left(\frac{\delta T}{T}\right)_0 = -\frac{1}{2}\left(\frac{\delta T}{T}\right)_{int}$$

Here, we have used the fact that perturbations (hence temperature fluctuations) are frozen outside the horizon (hence no $\sin\omega t$ term). For a given perturbation mode, the evolution in terms of a harmonic oscillator is as follows:



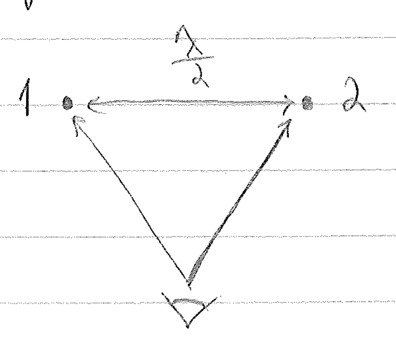


Since power is $\propto \left(\frac{\delta T}{T}\right)^2$, oscillations in temperature fluctuations give rise to:



T ≡ Period of oscillations

If the observed temperature anisotropy was due to oscillations in temperature fluctuations only, then power would be zero at $\frac{1}{4}$ period, $\frac{3}{4}$ period, ... However, this is not what we see in the CMB power spectrum. An important effect that lifts point with no power is the Doppler effect. Looking at a given perturbation mode in the sky, we observe temperature difference between two points separated by half a wavelength.

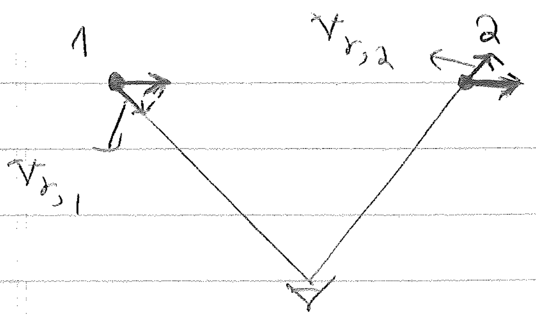


When the fluid in point 1 is compressed, it is because of rarefacti in point 2 and vice versa. Therefore points 1 and 2 have a 180° phase difference in their oscillatory motion. In fact, we have a standing wave in the fluid. The zeros in the power spectrum corresponds to the times when $\delta_B = 0$.

However, we note that the fluid is also moving back and forth between points 1 and 2 (sound wave is a longitudinal wave). Quantitatively, we have:

$$\delta_B = \delta_{B,0} \cos\omega t \quad , \quad \dot{\delta}_B = -\omega \delta_{B,0} \sin\omega t$$

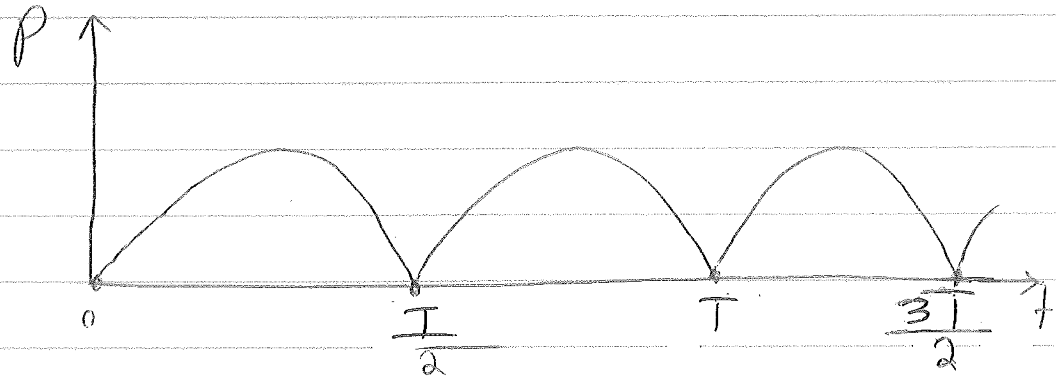
Therefore when $\cos\omega t = 0$, the velocity of the fluid at points 1 and 2 is maximum. Radial component of the fluid motion along the line of sight results in a Doppler shift in the energy (equivalently temperature) of photons that leave these points as follows:



$v_{r,1} < 0 \Rightarrow$ Blueshift

$v_{r,2} > 0 \Rightarrow$ Redshift

The Doppler effect then results in a contribution to $\left(\frac{\delta T}{T}\right)_{obs}$ which has $90°$ phase difference with oscillations in δ_B:



When combined, we have: